The equation of state for the solar interior

2006 J. Phys. A: Math. Gen. 39 4441

(http://iopscience.iop.org/0305-4470/39/17/S19)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **39** (2006) 4441–4446 doi:10.1088/0305-4470/39/17/S19

The equation of state for the solar interior

Werner Däppen[1]

International University Bremen, Campus Ring 1, 28759 Bremen, Germany

E-mail: dappen@usc.edu

Received 11 October 2005
Published 7 April 2006

Abstract
Helioseismology has become a very successful diagnosis of the equation of state. Although the gas in the solar interior is only weakly coupled and weakly degenerate, the great observational accuracy of the helioseismological measurements puts strong constraints on the nonideal part of the equation of state. For solar and stellar modelling, a high-quality equation of state is crucial. But the inverse is also true: the astrophysical data (helioseismic today, asteroseismic tomorrow) can put constraints on the physical formalisms, thus making the Sun and the stars laboratories for plasma physics.

PACS numbers: 51.30.+i, 96.60.Ly

1. Introduction

Since the early 1960s the surface of the Sun has been known to be in a regular pulsating motion with periods of about 5 min. Only in the 1970s it was recognized that these so-called solar oscillations are manifestations of *global motions* of the Sun about its equilibrium. Helioseismology is the name of the branch of astrophysics that deals with deciphering these data, which cover the whole range of spherical harmonics from $l = 0$ (radial) to very high angular order (above $l = 1000$). The observed solar oscillation modes are standing acoustic waves; hence the quantity most obviously probed is sound speed.

There are numerous introductions to helioseismology (see, for instance, Bahcall and Ulrich 1988, Gough 1993, Christensen-Dalsgaard *et al* 2000, Christensen-Dalsgaard 2002). In addition, there are reviews that specifically address the helioseismic determination of the equation of state (see Christensen-Dalsgaard and Däppen 1992, Baturin *et al* 2000). In contrast to denser objects (low-mass stars, white dwarfs, brown dwarfs, planets, or even neutron stars; see Chabrier 2005, Potekhin *et al* 2005), for the Sun's interior, a simple ideal-gas model of the plasma had been quite adequate before the advent of helioseismology. However, for the helioseismological accuracy of the early 1980s, a need to go beyond the

---

[1] Permanent Address: University of Southern California, Los Angeles, California, 90089-1342, USA.

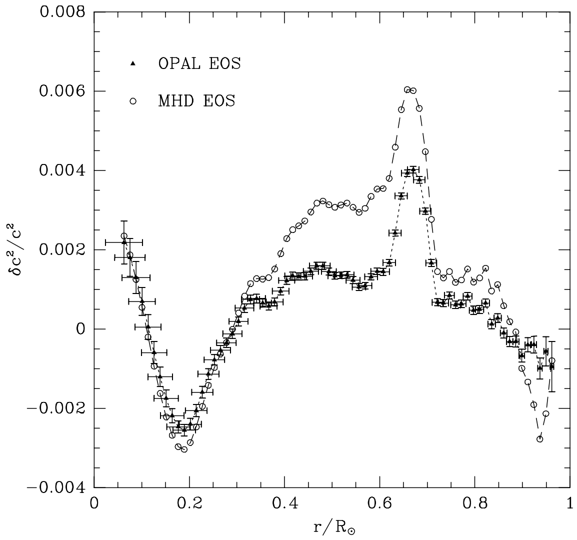

Figure 1. Difference between squared sound speed from inversion and two solar models. Figure by S Basu.

ideal-gas approximation was recognized (see Berthomieu *et al* 1980, Ulrich 1982, Noels *et al* 1984). Towards the end of the 1980s, with even better data, a clearer picture began to emerge. Christensen-Dalsgaard *et al* (1988, 1996) demonstrated that it was essential to include the Coulomb correction. Across the solar interior, the relative Coulomb pressure correction peaks twice, once in the outer part of the convection zone (about -8%) and once in the solar core (about -1%). For solar conditions, the Debye–Hückel (DH) theory is a good approximation for the leading term of the Coulomb correction.

Helioseismic equation-of-state studies use solar models based on sophisticated new equations of state. Particularly popular are those underlying the two major recent efforts to re-compute the opacity in stellar interiors. One of these efforts is the international Opacity Project (OP; see the books by Seaton 1995, Berrington 1997); it contains the so-called Mihalas–Hummer–Däppen equation of state (Hummer and Mihalas 1988, Mihalas *et al* 1988, Däppen *et al* 1988, Nayfonov *et al* 1999, Trampedach *et al* 2006); hereinafter MHD) and it deals with *heuristic* concepts about the modification of atoms and ions in a plasma. The other effort is being pursued at Lawrence Livermore National Laboratory by the OPAL group (Iglesias and Rogers 1996, Rogers *et al* 1996); its equation of state is based on a detailed *systematic* method to include density effects in a plasma (Rogers 1977, 1986, Rogers and Nayfonov 2002).

Although approximate asymptotic techniques (see Christensen-Dalsgaard *et al* 1985, Gough 1993) exist to invert solar oscillation frequencies for the internal sound speed, for an accurate analysis of the observations, a fully-fledged, non-asymptotic numerical treatment of the oscillations is mandatory (see Gough *et al* 1996). Figure 1 is a typical result of such a numerical inversion (Basu and Christensen-Dalsgaard 1997). It shows the relative difference (in the sense Sun–model) between the squared sound speed obtained from inversion of oscillation data and that of a two standard solar models. The two solar models used are identical in all respects except for their equation of state, MHD (circles) and OPAL (triangles), respectively. For the present purpose, we can consider inversion results such as figure 1 as the *data* of helioseismology, disregarding the procedure through which they were actually obtained from solar oscillation frequencies. The figure shows that overall models with the OPAL equation of state represent reality better than models with MHD.

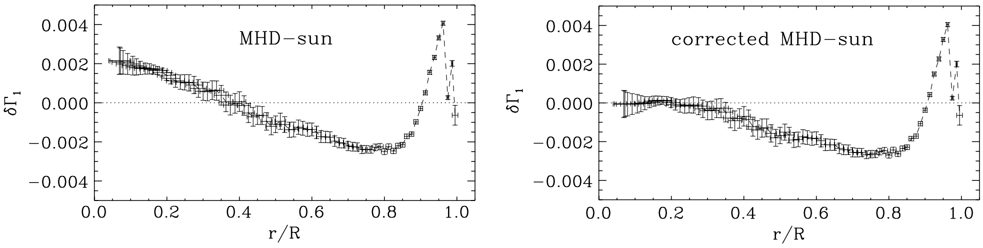

Figure 2. Difference between γ_1 (here denoted Γ_1) of a solar model and observation, for models with nonrelativistic electrons (left) and relativistic electrons (right). Figures by J Elliot.

2. Two examples of recent equation-of-state issues

2.1. Relativistic electrons

Figure 1 shows a discrepancy between theoretical and inverted values for both OPAL and MHD. It turned out that it was due to *relativistic* electrons. It happened that the original versions of MHD and OPAL only treated *non-relativistic* partially degenerate electrons. This was the cause of a discrepancy found in a recent helioseismic inversion for the adiabatic gradient $\gamma_1 = (\partial \ln p/\partial \ln \rho)_s$ (s being specific entropy) (Elliott and Kosovichev 1998).

The top panel of figure 2 shows this discrepancy for MHD. (A corresponding figure for OPAL would look essentially the same.) The relevant deviation occurs in the central 30% parts of the Sun. A relativistic treatment of the degenerate electrons in the solar model (bottom panel) removes the discrepancy nicely. As a result, upgrades to include relativistic electrons were since made both to MHD (Gong *et al* 2001a, 2001b) and OPAL (Rogers and Nayfonov 2002).

2.2. Effect of excited states in hydrogen and helium

Another effect beyond the Debye–Hückel correction is the signature of the internal partition functions. (Nayfonov and Däppen 1998) discovered a 'wiggle' in the thermodynamic quantities, located in the hydrogen and helium ionization zones. This effect, due to excited states, has probably already been observed in the Sun, because new observations (Basu *et al* 1999) suggest that in the top 2% of the solar radius, MHD models can give a more accurate match with the data than OPAL models. Since it turns out that in this region, the discrepancy between MHD and OPAL is essentially reflected by the aforementioned wiggle (Nayfonov and Däppen 1998), the result of the inversion (Basu *et al* 1999) could mean a validation of an MHD-like treatment (Hummer and Mihalas 1988) of exited states.

The main result of (Basu *et al* 1999) is shown in figure 3. It is the result from an inversion of observed solar oscillation frequencies for the *intrinsic* γ_1 difference between the Sun and a solar model. The intrinsic difference is that part of the γ_1 difference which is due to the difference in the equation of state itself; there is a further component to the γ_1 difference caused by the change to the structure of the solar model resulting from the difference in the equation of state (Basu and Christensen-Dalsgaard 1997). The error bars shown in figure 3 are based on combined errors of the inversion method and observational errors.

Figure 3 should not be over-interpreted, however, because present uncertainties in the inversion of the upper layers of the Sun (e.g., turbulent pressure, magnetic fields, nonlocal thermodynamic effect due to radiation, uncertainties in the chemical composition) preclude so far a definitive interpretation, and further clarifying work is in progress. In the slightly deeper

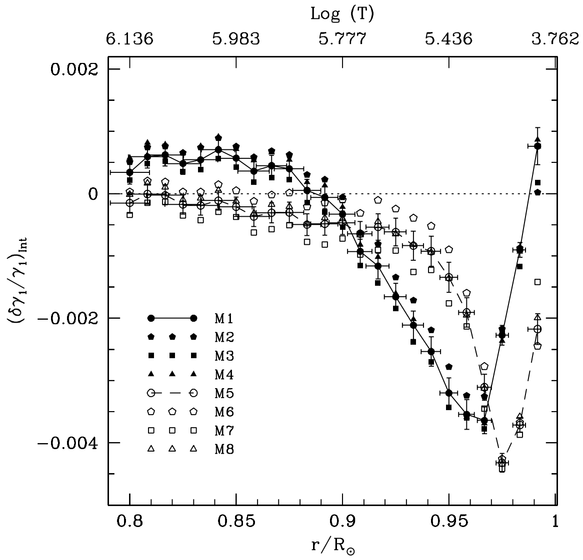

Figure 3. Intrinsic difference between γ_1 obtained from an inversion of helioseismological data (Basu *et al* 1999), and γ_1 of 4 MHD models (M1-4: filled points) and 4 OPAL models (M5-8: empty points), respectively. All results are in the sense 'Sun–model' (for more detail, see Basu *et al* 1999).

regions (below a depth of about 3% of the solar radius) the findings of the study (Basu *et al* 1999) are more reliable, and they confirm the findings of figure 1, that is, the overall OPAL is a better equation of state than MHD.

3. Directions for future developments

An equation of state more accurate than OPAL or MHD is still necessary. Although the equation of state, as evidenced by figures 1 and 3, is quite satisfactory, improvements are still needed. One day, perhaps, further systematic expansions of a physical-picture formalism, such as OPAL, can reach agreement with helioseismological observations, within the observational errors. However, such further expansions will be a formidable task, and the likelihood of success is difficult to estimate. The size of such an effort has to be measured by the standard of present-day physical-picture formalisms. So far, only one group (the OPAL group at Livermore) has realized a practically useful formalism that satisfies the exacting demands of solar modellers. However, their results are only available in the form of pre-computed tables (Rogers *et al* 1996, Rogers and Nayfonov 2002). Incidentally, the newest OPAL tables to be released later this year will allow modelling of stars with masses down to 0.1 M$_\odot$ (Rogers, private communication).

Since the OPAL computer code is still proprietary and belongs exclusively to the Livermore group, it makes sense to retrofit chemical-picture formalisms so that they coincide with OPAL. Liang and Däppen (2004) have successfully emulated the OPAL equation of state, for a simple hydrogen-only plasma under solar-envelope conditions. Of course, such efforts need not stop there: because of their heuristic nature, chemical-picture formalisms can, in principle, be tuned to mimic any other formalism, or direct observational results. An earlier attempt to simulate some aspects of the OPAL equation of state was made in the so-called

SIREFF equation of state (see Rogers *et al* 1996). It is based on the Eggleton *et al* (1973) (EFF) equation of state (for further details on SIREFF and EFF, see Däppen and Guzik 2000).

In order to get an equation of state that has the potential to be superior to OPAL (albeit less systematic and more intuitive), one can enrich chemical-picture formalisms with terms originating in the physical picture. The aforementioned OPAL emulator (Liang and Däppen 2004) shows a way. An alternative way was presented by Starostin *et al* (2003), Starostin and Roerich (2005). Their resulting solar models look promising (Baturin *et al* 2004, Gryaznov *et al* 2005).

4. Conclusions

While currently available equations of state give reasonable accuracy for solar modellers, the observational data have the potential to aim higher, not only for better solar models, but especially for a maximum use of the Sun to serve as a plasma-physics experiment. In a few years, such research will be greatly stimulated by the broad-range scientific applications of the large lasers currently under construction (NIF, Megajoule).

Acknowledgments

This work was supported by the grant AST-0307578 of the National Science Foundation.

References

Bahcall J N and Ulrich R K 1988 *Rev. Mod. Phys.* **60** 297

Basu S and Christensen-Dalsgaard J 1997 *Astron. Astrophys.* **322** L5

Basu S, Däppen W and Nayfonov A 1999 *Astrophys. J.* **518** 985

Baturin V A, Ajukov S V, Gryaznov V K, Iosilevskiy I L and Starostin A N 2004 in *Equation of State and Phase Transition Issues in Models of Ordinary Astrophysical Matter* ed V Celebonović, W Däppen and D Gough (*AIP Conf. Proc.* 731) (New York: Melville) p 178

Baturin V A, Däppen W, Gough D O and Vorontsov S V 2000 *Mon. Not. R. Astron. Soc.* **316** 71

Berrington K A 1997 *The Opacity Project* vol 2 (Bristol: Institute of Physics Publishing)

Berthomieu G, Cooper A J, Gough D O, Osaki Y, Provost J and Rocca A 1980 *Lecture Notes in Phys.* vol 125 ed H A Hill and W Dziembowski (Heidelberg: Springer) p 307

Chabrier G 2005 this volume

Christensen-Dalsgaard J 2002 *Rev. Mod. Phys.* **74** 1073–129

Christensen-Dalsgaard J and Däppen W 1992 *Astron. Astrophys. Rev.* **4** 267

Christensen-Dalsgaard J, Däppen W and Lebreton Y L 1988 *Nature* **336** 634

Christensen-Dalsgaard J, Gough D O and Toomre J 1985 *Science* **229** 923

Christensen-Dalsgaard J, Däppen W, Dziembowski W A and Guzik J A 2000 *Proc. Variable Stars as Important Astrophysical Tools* ed C Ibanoglu (Dordrecht: Kluwer) p 59–176

Christensen-Dalsgaard J, Däppen W and The GONG Team 1996 *Science* **272** 1286

Däppen W and Guzik J A 2000 *Proc. Variable Stars as Essential Astrophysical Tools* ed C Ibanoglu (Dordrecht: Kluwer) p 177

Däppen W, Mihalas D, Hummer D G and Mihalas B W 1988 *Astrophys. J.* **332** 261

Eggleton P P, Faulkner J and Flannery B P 1973 *Astron. Astrophys.* **23** 325

Elliott J R and Kosovichev A G 1998 *Astrophys. J.* **500** L199

Gong Z-G, Däppen W and Zejda L 2001a *Astrophys. J.* **546** 1178

Gong Z-G, Zejda L, Däppen W and Aparicio J M 2001b *Comp. Phys. Commun.* **136** 294

Gough D O, Kosovichev A G, Toomre J and the GONG Team 1996 *Science* **272** 1296

Gough D O 1993 in *Astrophysical Fluid Dynamics* ed J-P Zahn and J Zinn-Justin (Amsterdam: North-Holland) p 399–560

Gryaznov V K, Iosilevskiy I L, Starostin A N, Fortov V E, Ayukov S V and Baturin V A 2005 this volume

Hummer D G and Mihalas D 1988 *Astrophys. J.* **331** 794

Iglesias C A and Rogers F J 1996 *Astrophys. J.* **464** 943

Liang A and Däppen W 2004 in *Helio- and Asteroseismology: Towards a Golden Future, SOHO14–GONG2004* (Noordwijk, The Netherlands: ESA Publications Division) p 548

Mihalas D, Däppen W and Hummer D G 1988 *Astrophys. J.* **331** 815

Nayfonov A and Däppen W 1998 *Astrophys. J.* **499** 489

Nayfonov A, Däppen W, Hummer D G and Mihalas D M 1999 *Astrophys. J.* **526** 451–64

Noels A, Scuflaire R and Gabriel M 1984 *Astron. Astrophys.* **130** 389

Potekhin A Y, Chabrier G, Lai D, HO W C G and van Adelsberg M 2005 this volume

Rogers F J 1977 *Phys. Lett.* A **61** 358

Rogers F J 1986 *Astrophys. J.* **310** 723–8

Rogers F J private communication

Rogers F J, Swenson F J and Iglesias C A 1996 *Astrophys. J.* **456** 902

Rogers F J and Nayfonov A 2002 *Astrophys. J.* **576** 1064

Seaton M J 1995 *The Opacity Project* vol 1 (Bristol: Institute of Physics Publishing)

Starostin A, Roerich V C and More R M 2003 *Contrib. Plasma Phys.* **43** 369

Starostin A and Roerich V C 2005 this volume

Trampedach R, Baturin V A and Däppen W 2006 *Astrophys. J.* in preparation

Ulrich R K 1982 *Astrophys. J.* **258** 404